

# Corrective Term in Wall Slip Equations for Knudsen Layer

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## Introduction

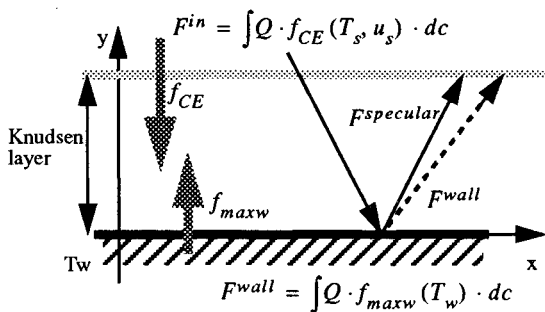
**A** RE-ENTRY vehicle along its trajectory through the atmosphere encounters various aerodynamics regimes with decreasing altitude: rarefied regime (low gas density for high altitude), transition, and continuum (low altitude).

The transition regime is characterized by slip conditions on the vehicle wall. To compute this kind of flowfield, the first approach is to couple continuum method and direct simulation. The second consists of using slip conditions as boundary conditions. In resuming the Gupta et al.<sup>1</sup> and Grad<sup>2</sup> calculations for a monoatomic gas, we found errors. The corrections lead to a supplementary term in the temperature jump. We will deal here with the simple monodimensional flow problem between two parallel plates.

## Classical Results

### Gupta et al.<sup>1</sup> Method (Half-Flux Approach)

The slip conditions are taken to exist across a thin Knudsen layer, which is on the order of one mean-free-path in thickness as sketched in the following figure. The assumption is to consider that variations of the velocity distribution function through the Knudsen layer are small. This implies constant fluxes  $F$  of conservative quantities  $Q$  (mass, momentum, and energy) across the Knudsen layer. At the upper boundary, we impose the Chapman–Enskog distribution  $f_{CE}$ , while the mechanism of reflection from surface uses accommodation coefficient  $\theta$ , with the Maxwellian distribution  $f_{maxw}$ .



### Grad Method<sup>2</sup>

Although the assumption is the same as in the precedent method, the Grad approach is a more mathematical one. It consists of writing the boundary condition

$$f^+(c_x, c_y, c_z) = \theta \cdot f^-(c_x, -c_y, c_z) + k \cdot e^{-\beta^2 c^2} \quad (c_y > 0)$$

with  $f^+$  distribution function associated to reflected particles,  $f^-$  the incident ones, and employing the Chapman–Enskog distribution for  $f^{+/-}$ .

In both cases, they obtain the following expressions:

$$u_s = \sqrt{\pi} \frac{2 - \theta}{2\theta} \cdot \frac{1}{p_s \beta_s} \cdot \frac{\partial u_s}{\partial y} \quad (\beta = \sqrt{m/2kT})$$

$$T_s = T_w - \frac{2 - \theta}{2\theta} \cdot \frac{\beta_s T_s \sqrt{\pi}}{p_s} \cdot q_y$$

where  $q_y$  is the  $y$  component of heat flux.

## Corrections

The errors are not the same in the two previous methods. In the first one, the Chapman–Enskog distribution (incident and specular reflection) and the Maxwellian (accommodation) are centered around 0, and the quantities  $Q$  for the fluxes are as follows:

velocity jump (tangential component momentum flux)

$$Q = mc_x \quad \text{at } y = 0$$

$$Qmc'_x = m(c_x - u_s) \quad \text{at } y = s$$

temperature jump (energy flux)

$$Q = \frac{1}{2} mc^2 \quad \text{at } y = 0 \quad \text{and} \quad Q = \frac{1}{2} mc^2 \quad \text{at } y = s$$

To be coherent, we must use the same reference, which is not the case at  $y = s$ . To correct this, we can e.g., express all the fluxes in terms of particle (and not peculiar) velocity, and center the Chapman–Enskog distribution around the slip unknowns and the Maxwellian around 0.

In Grad, it's a calculus error: in the expression [Ref. 2; Eq. (6.29), p. 383], we find that a  $\frac{1}{2}$  factor has been omitted (here,  $\alpha = 1 - \theta$ ):

$$\left[ \left( \frac{2\pi}{RT} \right)^{1/2} \frac{q_y}{p} \right]_s + \frac{8(1 - \alpha)}{1 + \alpha} \left[ 1 - \frac{T_w}{T} + \frac{p_{yy}}{2p} \left( \frac{3}{2} - \frac{T_w}{T} \right) - \left( 1 + \frac{p_{yy}}{2p} \right) \frac{u^2}{4RT} \right]_s = 0$$

With these corrections made, they both lead to the same corrected expression: the slip velocity appears in the temperature jump which becomes

$$T_s = T_w + \underbrace{\frac{1}{4R} u_s^2} - \frac{2 - \theta}{2\theta} \cdot \frac{\beta_s T_s \sqrt{\pi}}{p_s} \cdot q_y$$

## Remarks

In others works<sup>3,5</sup> this velocity term doesn't appear. It is due to the fact that slip equations were not studied with thermal and velocity gradients together. For example, Welander,<sup>3</sup> who resolves the Boltzmann equation in the Knudsen layer, deals with the temperature jump without macroscopic velocity, because of mathematical difficulties.

The coefficient  $1/(4R)$  being of the thousandth-order (gas constant  $R = 287 \text{ J/kg/K}$  for the air),  $u_s$  must be near 100 m/s to increase the temperature jump of only 10 K. Finally, the influence of this term becomes important for high slip velocity.

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### References

<sup>1</sup>Gupta, R. N., Scott, C. D., and Moss, J. N., "Slip-Boundary Equations for Multicomponent Nonequilibrium Airflow," NASA-CR-181252, April 1985; see also "Surface Slip Equations for Low-Reynolds-Number Multicomponent Gas Flows," AIAA Paper 84-1732, June 1984.

<sup>2</sup>Grad, H., "On the Kinetic Theory of Rarefied Gases," *Communications on Pure and Applied Mathematics*, Vol. 2, No. 4, 1949,

pp. 331–407; see also "Principles of the Kinetic Theory of Gases," *Encyclopedia of Physics*, edited by S. Fluegge, Vol. 12, Springer-Verlag, Berlin, 1958, pp. 205–293.

<sup>3</sup>Welander, P., "On the Temperature Jump in a Rarefied Gas," *Arkiv för Fysik*, Vol. 7 No. 44, 1954, pp. 507–553.

<sup>4</sup>Kennard, E. H., "Kinetic Theory of Gases," McGraw-Hill, New York, 1938.

<sup>5</sup>Kogan, M. N., "Rarefied Gas Dynamics," Plenum, New York, 1969.

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