Corrective Term in Wall Slip Equations for Knudsen Layer

Rémy Goniak* and Georges Duffat†
Commissariat à l'Energie Atomique,
33114 Le Barp, France

Introduction

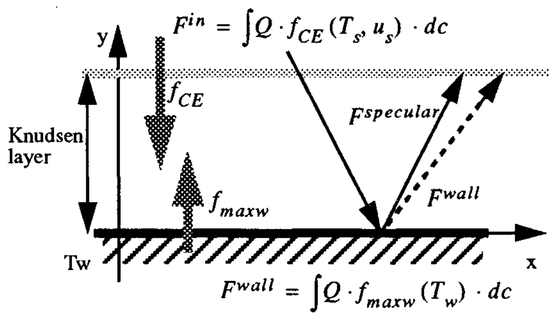
A RE-ENTRY vehicle along its trajectory through the atmosphere encounters various aerodynamics regimes with decreasing altitude: rarefied regime (low gas density for high altitude), transition, and continuum (low altitude).

The transition regime is characterized by slip conditions on the vehicle wall. To compute this kind of flowfield, the first approach is to couple continuum method and direct simulation. The second consists of using slip conditions as boundary conditions. In resuming the Gupta et al.[1] and Grad[2] calculations for a monoatomic gas, we found errors. The corrections lead to a supplementary term in the temperature jump. We will deal here with the simple monodimensional flow problem between two parallel plates.

Classical Results

Gupta et al.[1] Method (Half-Flux Approach)

The slip conditions are taken to exist across a thin Knudsen layer, which is on the order of one mean-free-path in thickness as sketched in the following figure. The assumption is to consider that variations of the velocity distribution function through the Knudsen layer are small. This implies constant fluxes F of conservative quantities Q (mass, momentum, and energy) across the Knudsen layer. At the upper boundary, we impose the Chapman–Enskog distribution f_{CE}, while the mechanism of reflection from surface uses accommodation coefficient θ, with the Maxwellian distribution f_{maxw}.



Grad Method[2]

Although the assumption is the same as in the precedent method, the Grad approach is a more mathematical one. It consists of writing the boundary condition

$$f^+(c_x, c_y, c_z) = \theta \cdot f^-(c_x, -c_y, c_z) + k \cdot e^{-\beta^2 c^2} \qquad (c_y > 0)$$

with f^+ distribution function associated to reflected particles, f^- the incident ones, and employing the Chapman–Enskog distribution for $f^{+/-}$.

In both cases, they obtain the following expressions:

$$u_s = \sqrt{\pi}\,\frac{2-\theta}{2\theta}\cdot\frac{1}{p_s\beta_s}\cdot\frac{\partial u_s}{\partial y} \qquad (\beta = \sqrt{m/2kT})$$

$$T_s = T_w - \frac{2-\theta}{2\theta}\cdot\frac{\beta_s T_s\sqrt{\pi}}{p_s}\cdot q_y$$

where q_y is the y component of heat flux.

Corrections

The errors are not the same in the two previous methods. In the first one, the Chapman–Enskog distribution (incident and speculary reflection) and the Maxwellian (accommodation) are centered around 0, and the quantities Q for the fluxes are as follows:

velocity jump (tangential component momentum flux)

$$Q = mc_x \quad \text{at} \quad y = 0$$

$$Qmc'_x = m(c_x - u_s) \quad \text{at} \quad y = s$$

temperature jump (energy flux)

$$Q = \tfrac{1}{2}mc^2 \quad \text{at} \quad y = 0 \quad \text{and} \quad Q = \tfrac{1}{2}mc^2 \quad \text{at} \quad y = s$$

To be coherent, we must use the same reference, which is not the case at $y = s$. To correct this, we can e.g., express all the fluxes in terms of particle (and not peculiar) velocity, and center the Chapman–Enskog distribution around the slip unknowns and the Maxwellian around 0.

In Grad, it's a calculus error: in the expression [Ref. 2; Eq. (6.29), p. 383], we find that a $\frac{1}{4}$ factor has been omitted (here, $\alpha = 1 - \theta$):

$$\left[\left(\frac{2\pi}{RT}\right)^{1/2}\frac{q_y}{p}\right]_s + \frac{8(1-\alpha)}{1+\alpha}\left[1 - \frac{T_w}{T} + \frac{p_{yy}}{2p}\left(\frac{3}{2} - \frac{T_w}{T}\right) - \left(1 + \frac{p_{yy}}{2p}\right)\frac{u^2}{4RT}\right]_s = 0$$

With these corrections made, they both lead to the same corrected expression: the slip velocity appears in the temperature jump which becomes

$$T_s = T_w + \underbrace{\frac{1}{4R}}\,u_s^2 - \frac{2-\theta}{2\theta}\cdot\frac{\beta_s T_s\sqrt{\pi}}{p_s}\cdot q_y$$

Remarks

In others works[3-5] this velocity term doesn't appear. It is due to the fact that slip equations were not studied with thermal and velocity gradients together. For example, Welander,[3] who resolves the Boltzmann equation in the Knudsen layer, deals with the temperature jump without macroscopic velocity, because of mathematical difficulties.

The coefficient $1/(4R)$ being of the thousandth-order (gas constant $R = 287$ J/kg/K for the air), u_s must be near 100 m/s to increase the temperature jump of only 10 K. Finally, the influence of this term becomes important for high slip velocity.

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*Research Engineer, CEA/CESTA BP n° 2.
†Senior Scientist, CEA/CESTA BP n° 2.

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